

Bulk and shear viscosities of hot and dense hadron gas

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We estimate bulk and shear viscosity at finite temperature and baryon densities of hadronic matter within hadron resonance gas model. For bulk viscosity we use low energy theorems of QCD for the energy momentum tensor correlators. For shear viscosity coefficient, we estimate the same using molecular kinetic theory to relate the shear viscosity coefficient to average momentum of the hadrons in the hot and dense hadron gas. The bulk viscosity to entropy ratio increases with chemical potential and is related to the reduction of velocity of sound at nonzero chemical potential. The shear viscosity to entropy ratio on the other hand, shows a nontrivial behavior with the ratio decreasing with chemical potential for small temperatures but increasing with chemical potential at high temperatures and is related to decrease of entropy density with chemical potential at high temperature due to finite volume of the hadrons.

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I. INTRODUCTION

Recently, the transport properties of hot and dense matter has attracted lot of attention in the context of relativistic heavy ion collisions[1] as well as cosmology[2]. Such properties enter in the hydrodynamical evolution and therefore essential for studying the near equilibrium evolution of a thermodynamic system. In the context of heavy ion collisions, the coefficients of shear viscosity perhaps has been the mostly studied transport coefficient. The spatial anisotropy in a nuclear collision gets converted to a momentum anisotropy through a hydrodynamic evolution. The equilibration of momentum anisotropy is mainly controlled by shear viscosity. Indeed, elliptic flow measurement at RHIC led to $\frac{\eta}{s}$, the ratio of shear viscosity (η) to the entropy density s , close to $\frac{1}{4\pi}$ which is the smallest for any known liquid in nature [3]. Indeed, arguments based on ADS/CFT correspondence suggest that the ratio $\frac{\eta}{s}$ cannot be lower than this 'Kovtun-Son-Starinets' (KSS) bound [4]. Thus the quark gluon plasma(QGP) formed in the heavy ion collision is the most perfect fluid.

Apart from shear viscosity, the transport coefficient that relates the momentum flux with a velocity gradient is the bulk viscosity. Generally, it was believed that the bulk viscosity does not play any significant role in the hydrodynamic evolution of the matter produced in heavy ion collision experiments. The argument being that the bulk viscosity ζ scales like $\epsilon - 3p$ and therefore will not play any significant role as the matter might be following the ideal gas equation of state. However, in course of the expansion of the fire ball the temperature can be near the critical temperature T_c where $\epsilon - 3p$ can be large as expected from the lattice QCD simulations[5, 6] leading to large value of bulk viscosity. This in turn can give rise to phenomena of cavitation when the pressure vanishes and the hydrodynamic description for the evolution breaks down[7].

There has been various attempts to estimate bulk viscosity for strongly interacting matter. The rise of bulk viscosity near the transition temperature has been observed in various effective models of strong interaction. These include chiral perturbation theory[8], quasi particle models [9] as well as Nambu-Jona-Lasinio model [10]. One of the interesting way to extract this is using symmetry properties of QCD once one realizes that the bulk viscosity characterizes the response to conformal transformation. This was attempted in Ref.[11]. Based on Kubo formula for the ζ and the low energy theorems [12] bulk viscosity gets related to thermodynamic properties of strongly interacting system.

It may be noted that, it is also of both practical and fundamental importance to know the transport coefficients in the hadron phase to distinguish the signatures of QGP matter and hadronic matter. The computation of the transport coefficient of the hadronic mixture is not an easy task. There have been various attempt on this field over last few years involving various approximations like relaxation time approximation, Chapman-Encskog as well as Green Kubo approach to estimate the shear viscosity to entropy ratio using different effective models for hadronic interactions [10, 13–16].

In a different approach, $\frac{\eta}{s}$ has also been calculated within a hadron resonance gas model in an excluded volume approximation [17] with a molecular kinetic theory approach to relate shear viscosity coefficient to the average momentum transfer. This was used later to include the effects of rapidly rising hadronic density of states near the critical temperature modeled by hagedorn type exponential rise of density of states [18]. Such a description could describe the lattice data and indicated that the hadronic matter could become almost a perfect fluid where $\frac{\eta}{s}$ could approach the KKS bound. Later lattice data [6] which indicated a lower pseudocritical temperature about 160 MeV led to the assertion that the hot hadronic matter described through hadron resonance gas is far from being a perfect fluid[20]. All these studies have been done at zero baryon density.

It has been also known that the basic features of hadronization in heavy ion collisions are well described by the hadron resonance gas models. The multiplicities of particle abundances of various hadrons in these experiments show good agreement with the corresponding thermal abundances calculated in HRG model with appropriately chosen temperature and chemical potentials [19]. In the present work, we generalize the above approach of [20] for studying viscosity coefficients within the ambit of hadron resonance gas model to include finite chemical potential effects. This can possibly have some relevance on the current and planned experiments with heavy ion collisions at *beam energy scan* at RHIC [21], *compressed baryonic matter* at GSI [22] and *neutclotron-based ion collider facility (NICA)* at Dubna [23].

The shear viscosity to entropy ratio at finite baryon density has been estimated using relativistic Boltzmann equations for pion nucleon system using phenomenological scattering amplitude[24, 25]. This leads to the ratio as a decreasing function of chemical potential in the T- μ plane. Further, this ratio as a function of chemical potential shows a valley structure at low temperature which was interpreted as a signature of liquid gas phase transition[24, 25].

The bulk viscosity at finite chemical potential using low energy theorems of QCD has been studied in Ref.[26]. This was estimated using a Schwinger Dyson approach to calculate the dressed quark propagator at finite chemical

potential to use it for calculation of thermodynamical quantities needed to estimate bulk viscosity. As mentioned, we shall estimate these viscosity coefficients within the ambit of hadron resonance gas which can be a complimentary to the above approaches.

We organize the paper as follows. In the following section we recapitulate the results of Ref.[11] for bulk viscosity coefficient as related to the thermodynamic quantities using Kubo formula and low energy theorems generalized to include finite chemical potential terms. We also note down here the expression for the shear viscosity using quantum molecular dynamics method modified appropriately for relativistic system. In section III we spell out the hadron resonance gas model including a hagedorn spectrum above a cut off and the resulting thermodynamics. We estimate the quark condensates in a thermal, dense medium of hadron gas in a subsection here. In section IV we discuss the results. Finally, we summarize and conclude in section V.

II. BULK AND SHEAR VISCOSITY COEFFICIENTS AT FINITE T AND μ

Bulk viscosity corresponds to the response of the system to conformal transformations and can be written as per Kubo formula as a bilocal correlation function[11]

$$\zeta = \lim_{\omega \rightarrow 0} \frac{1}{9\omega} \int_0^\infty dt \int d\mathbf{x} \exp(i\omega t) [\theta_\mu^\mu(x), \theta_\mu^\mu(0)] \equiv \int d^4x iG^R(x) \quad (1)$$

with $G^R(x)$ being the retarded function for the trace of energy momentum tensor. One can introduce a spectral function $\rho(\omega, \mathbf{p}) = -(1/\pi)ImG(\omega, \mathbf{p})$ to write a dispersion relation for the $G^R(\omega, \mathbf{p})$. Assuming an ansatz for the spectral function at low energy[11] as $\rho(\omega, \mathbf{0})/\omega = (9\zeta/\pi)(\omega_0^2/(\omega_0^2 + \omega^2))$, where, ω_0 is a scale at which perturbation theory becomes valid, the bulk viscosity can be written as

$$9\zeta\omega_0 = 2 \int_0^\infty du \frac{\rho(u, 0)}{u} du = \int d^4x \langle \theta_\mu^\mu(x) \theta_\mu^\mu(0) \rangle \equiv \Pi \quad (2)$$

The stress energy tensor for QCD is given as

$$\theta_\mu^\mu = m\bar{q}q + \frac{\beta(g)}{2g} G_{\mu\nu}^a G^{a\mu\nu} \equiv \theta_q + \theta_g \quad (3)$$

In the above g is the strong coupling and $\beta(g)$ is the QCD beta function that decides the running of the QCD coupling. Thus the evaluation of the bulk viscosity reduces to evaluation of the stress energy correlator. This is done by using the low energy theorems of QCD generalized to finite temperature and density according to which for any operator \hat{O} , its correlator with the gluonic part of the stress tensor θ_g is given as

$$\int d^4x \langle \theta_g(x) \hat{O} \rangle = (\hat{D} - d) \langle \hat{O} \rangle(T, \mu), \quad (4)$$

where, $\hat{D} = T\partial/\partial T + \mu\partial/\partial\mu - d$, with d being the canonical dimension of the operator \hat{O} . Using Eq.(4) in Eq.(2) one has

$$\begin{aligned} \Pi &= (\hat{D} - 4) \langle \theta_\mu^\mu \rangle + (\hat{D} - 2) \langle \theta^{q\mu}_\mu \rangle \\ &= 16|\epsilon_{vac}^q| + 6(f_\pi^2 m_\pi^2 + f_k^2 m_k^2) \\ &\quad + TS\left(\frac{1}{c_s^2} - 3\right) + \left(\mu \frac{\partial}{\partial\mu} - 4\right)(\epsilon^* - 3p^*) + (\hat{D} - 2)m_q \langle \bar{q}q \rangle_* \end{aligned} \quad (5)$$

In the above we have used $\langle \theta_\mu^\mu \rangle = \epsilon - 3p$ and the thermodynamic relations $c_v = \partial\epsilon/\partial T$, $\partial p/\partial T = s$ and $c_s^2 = S/c_v$ for the velocity of sound of the medium. We have also separated the contributions to the correlators in terms of the vacuum and the medium. In Eq.(5) we have neglected terms quadratic in the current quark masses and have used PCAC relations to express vacuum condensates to the masses and decay widths of pions and kaons. It is trivial to check that for $\mu = 0$ Eq.(5) reduces to the main results of Ref.[11]. For $T=0$ and $\mu \neq 0$, one can simplify Eq.(5) and Eq.(2) reduces to

$$9\zeta(\mu)\omega_0 = 16P(\mu) - 7\mu\rho + \mu^2 \frac{\partial\rho}{\partial\mu} + \left(\mu \frac{\partial}{\partial\mu} - 2\right)m \langle \bar{q}q \rangle \quad (6)$$

We might note here that the above expression differs from the same given in Ref.[26]. This, however, matches with the expression given in Ref.[27] in the appropriate limit, where, bulk viscosity was computed including the effects of magnetic field at finite baryon densities and temperature.

Thus the coefficients of bulk viscosity gets related to the vacuum properties of QCD as well as to the equilibrium thermodynamic system parameters of QCD like the velocity of sound, non-ideality and the in medium quark condensates. These thermodynamic quantities shall be estimated within hadron resonance gas model which we shall spell out in the next section.

Next, we consider the shear viscosity coefficient η for the hadronic medium. It is known that hadrons interact in various channels and there is possibility of attractive and repulsive interactions. Within the hadron resonance model, the attractive channels are effectively included by including the resonances and the repulsive channels can be modeled in a simple manner through and excluded volume correction [28–30]. The shear viscosity in a relativistic gas of multi component hard core spheres can be written as [17, 20]

$$\eta = \frac{5}{64\sqrt{8}r^2} \sum_i \langle |\mathbf{p}| \rangle \frac{n_i}{n} \quad (7)$$

where, $\langle |\mathbf{p}| \rangle$ is the average momentum of the i -th species particles and r corresponds to hard core radius of each hadron. Further, in the above, n_i is the number density of the i -th particle species and $n = \sum_i n_i$.

III. HADRON RESONANCE GAS MODEL

The central quantity in the hadron resonance gas models (HRGM) is the thermodynamic potential which is that of a free boson or fermion gas and is given as

$$\log(Z, \beta, \mu,) = \int dm (\rho_M(m) \log Z_b(m, v, \beta, \mu) + \rho_B(m) \log Z_f(m, v, \beta, \mu)) \quad (8)$$

where, the gas of hadrons is contained in a volume V , at a temperature β^{-1} and chemical potential μ . Z_b, Z_f are the partition functions of boson and fermions respectively with mass m . Further, ρ_M and ρ_B are the spectral densities of bosons and fermions respectively. Using Eq.(8), one can calculate the energy density ϵ by taking derivative with respect to β , pressure p , by taking a derivative with respect to V , number density ρ by taking a derivative with respect to μ . One can also find out the trace anomaly $\epsilon - 3p$, entropy density, specific heat as well as the speed of sound from these quantities.

Hadron properties enter in these models through the spectral densities $\rho_{B/M}(m)$. One common approach in HRGMs is taking all the hadrons and their resonances up to a mass cutoff Λ and write

$$\rho_{B/M}(m) = \sum_i^{M_i < \Lambda} g_i \delta(m - M_i) \quad (9)$$

where, the sum is over all the baryons or meson states up to a mass that is less than the cut off Λ . M_i are the masses of the known hadrons and g_i is the degeneracy factor (spin, isospin). On the other hand, an exponentially increasing density of state was necessary to explain the rapid increase in entropy density near the transition region in lattice QCD simulation [31]. Such exponential rise of density of states has also been used to study observables like dilepton production [32] as well as chemical equilibration [33]. Motivated by such observations we take the modified spectral function as [18, 34, 35]

$$\rho_{B/M}(m) = \sum_i^{M_i < \Lambda} g_i \delta(m - M_i) + \rho_{HS}(m) \quad (10)$$

where $\rho_{HS}(m)$ is the spectral density for the heavier Hagedorn states (HS). To describe the much needed large density of states, one can take an exponentially rising density of state [36] for ρ_{HS} beyond the cut-off Λ which implies an underlying string picture for hadrons. On the other hand, one can also consider a simple power law form introduced in Ref.[38] as a nice alternative to describe the rise of the hadronic mass spectrum [38]. We shall consider here both the forms for the continuum part of the spectral density given as

$$\rho_{exp} = \frac{A}{(m^2 + m_0^2)^2} e^{\frac{m}{T_H}} \quad (11)$$

$$\rho_{power} = \frac{A}{T_H} \left(\frac{m}{T_H} \right)^\alpha \quad (12)$$

where parametrization of the two spectral forms is given in table below.

spectral density	$T_H(GeV)$	A	$m_0(GeV)$	α
ρ_{exp}	0.210	0.63	0.5	-
ρ_{power}	0.180	0.51	-	3

We have taken the parameters A and m_0 for ρ_{exp} as in Ref. [31] and taken a different value for T_H so as to fit the lattice data of Ref.[39]. Similarly the parameters α and T_H for ρ_{power} is taken so as to fit the lattice data of Ref.[39] while keeping the parameter A same as taken in Ref.[20].

With the ansatz for the spectral densities, the pressure $P = P_M + P_B$ arising from mesons and baryons respectively are given by

$$P_M = \frac{1}{2\pi^2} \left[- \sum_i g_i \int k^2 dk \log(1 - \exp(-\beta\epsilon_i)) + \int_\Lambda^\infty \rho_{HS}(m) dm \frac{m^2}{\beta^2} K_2(\beta m) \right] \quad (13)$$

$$P_B = \frac{1}{2\pi^2} \left[- \sum_i g_i \int k^2 dk \left(\log(1 - \exp(-\beta(\epsilon_i - \mu))) + \log(1 - \exp(-\beta(\epsilon_i + \mu))) \right) + 2 \int_\Lambda^\infty \rho_H(m) dm \frac{m^2}{\beta^2} K_2(\beta m) \cosh(\beta\mu) \right] \quad (14)$$

Here, $K_n(x)$ is the modified Bessel function of order n . Similarly, the energy density $\epsilon = -\frac{1}{\beta} \frac{\partial}{\partial \beta}(\beta p) + \mu \frac{\partial}{\partial \mu} p = \epsilon_M + \epsilon_B$, with the energy density of mesons ϵ_M given as

$$\epsilon_M = \frac{1}{2\pi^2} \left[\sum_i g_i \int k^2 dk \frac{\epsilon_i}{\exp(\beta\epsilon_i) - 1} + \int_\Lambda^\infty \rho_{HS}(m) dm m^4 \left(\frac{3}{\beta^2 m^2} K_2(\beta m) + \frac{1}{\beta m} K_1(\beta m) \right) \right] \quad (15)$$

and, the contribution of the baryons to the energy density ϵ_B is given as

$$\epsilon_B = \frac{1}{2\pi^2} \left[\sum_i g_i \int k^2 dk \epsilon_i \left(\frac{1}{\exp(\beta(\epsilon_i - \mu)) + 1} + \frac{1}{\exp(\beta(\epsilon_i + \mu)) + 1} \right) + \int_\Lambda^\infty \rho_H(m) dm m^4 \left(\frac{3}{\beta^2 m^2} K_2(\beta m) + \frac{1}{\beta m} K_1(\beta m) \right) \right] \quad (16)$$

The baryon number density is given by

$$n_B = \frac{1}{2\pi^2} \left[g_i \int k^2 dk \left(\frac{1}{\exp(\beta(\epsilon_i - \mu)) + 1} - \frac{1}{\exp(\beta(\epsilon_i + \mu)) + 1} \right) + 2 \int_\Lambda^\infty \rho_H(m) dm \frac{m^2}{\beta^2} K_2(\beta m) \right] \quad (17)$$

Using these quantities one can calculate the other quantities like the interaction measure $\epsilon - 3p$, entropy density $s = \left(\frac{\partial p}{\partial T} \right)$ as needed for the estimation of bulk viscosity.

A. quark condensates in the hadronic medium

The other quantity we need to know is the quark condensates in the medium to estimate the bulk viscosity. To estimate this within the framework of HRGM, it is necessary to know the dependence of hadron masses on the current quark masses. The chiral condensate is given in terms of the thermodynamic potential (negative of the pressure) as $\langle \bar{q}q \rangle = -\frac{\partial p}{\partial m_q}$ which leads to

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 + \sum_{mesons} \frac{\sigma^M}{m_q} n_M + \sum_{baryons} \frac{\sigma^B}{m_q} n_B \quad (18)$$

where n_M and n_B are the scalar densities of mesons and baryons given respectively as

$$n_M = \frac{g_i}{2\pi^2} \int k^2 dk \frac{m_M}{\epsilon_M} \frac{1}{\exp(\beta\epsilon_M) - 1}, \quad (19)$$

$$n_B = \frac{g_i}{2\pi^2} \int k^2 dk \frac{m_B}{\epsilon_B} \left(\frac{1}{\exp(\beta(\epsilon_B - \mu_B)) + 1} + \frac{1}{\exp(\beta(\epsilon_B + \mu_B)) + 1} \right) \quad (20)$$

Further, the $\sigma^{M/B}$ is the hadronic sigma term i.e. the response of hadronic masses to the changes of the current quark masses

$$\sigma_q^{M/B} = m_q \frac{\partial M_{M/B}}{\partial m_q} \quad (21)$$

Thus computing the behavior of in-medium condensate within HRGM reduces to the problem of calculating the σ -terms of the hadrons. We do this in a manner similar to given in Ref.[40]. For the pseudoscalar bosons, we use the Gell Mann-Oakes-Renner (GOR) relation to have

$$\frac{\partial m_\pi^2}{\partial m_q} = -\frac{\langle \bar{q}q \rangle_0}{f_\pi^2} \left(1 + 2\kappa \frac{m_\pi^2}{f_\pi^2} \right) \quad (22)$$

$$\frac{\partial m_K^2}{\partial m_{q,s}} = -\frac{\langle \bar{q}q \rangle_0 + \langle \bar{s}s \rangle_0}{2f_K^2} \left(1 + 2\kappa \frac{m_K^2}{f_\pi^2} \right) \quad (23)$$

with the parameter $\kappa = 0.021 \pm 0.008$ [41]. here, we have taken $m_q = m_u = m_d = 5.5 \text{ MeV}$, $m_s = 138 \text{ MeV}$, $f_\pi = 92.4 \text{ MeV}$, $f_K = 113 \text{ MeV}$ $\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = \langle \bar{q}q \rangle_0 = (-240 \text{ MeV})^3$, $\langle \bar{s}s \rangle_0 = 0.8 \langle \bar{q}q \rangle_0$. For the other hadrons we use a model based on valence quark structure as in Ref. [42]. Here the masses of the baryons (B) or mesons (M) scale as

$$m_B = (3 - N_s)M_q + N_s M_s + \kappa_B \quad (24)$$

$$m_M = (2 - n_s)M_q + N_s M_s + \kappa_m \quad (25)$$

In the above, M_q , M_s are the constituent quark masses for the light and strange quarks respectively, $\kappa_{B/M}$ are constants depending upon the hadronic state but not on current quark masses and N_s is the numbers of strange quarks. The constituent quarks M_q and M_s partially account for the strong interaction dynamics. For computation of the σ term one needs to know the variation of the constituent quarks with current quark masses. This dependence is taken from Nambu-Jona-Lasinio model [43] where, the dynamical mass changes by 14 MeV as the current quark mass is changed from 0 to 5.5 MeV. Similarly for strange quark the mass change is about 235.5 MeV as current quark mass is varied from 0 to 140.7 MeV. This e.g. results in σ terms for nucleons and Λ hyperon as 42 MeV and 263.5 MeV respectively.

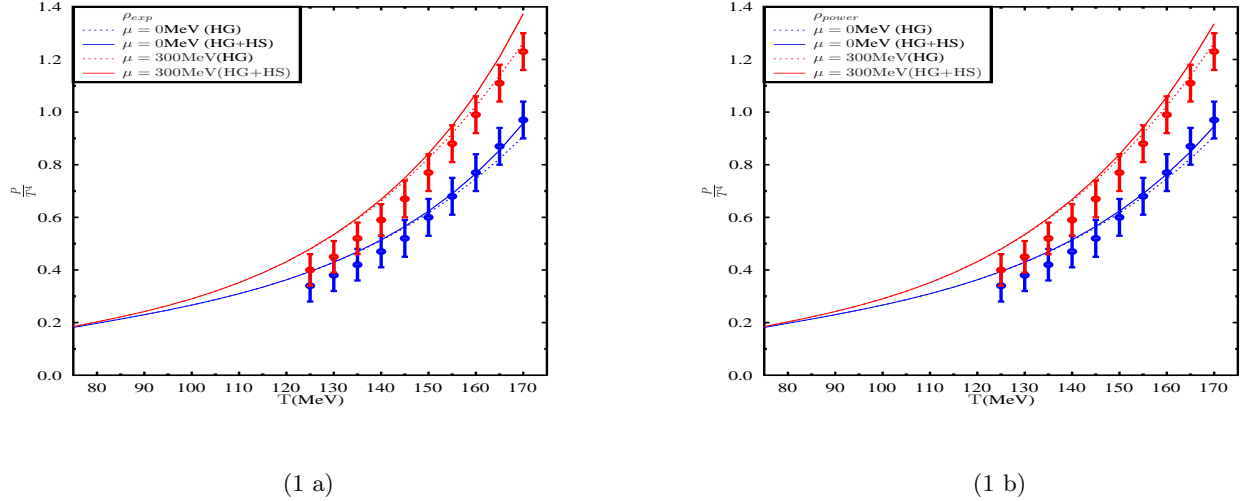


FIG. 1. Thermodynamics of hadron resonance gas. Left panel (Fig. 1 a) shows pressure as a function of temperature for $\mu_b = 0$ (blue) and $\mu_B = 300$ MeV (red) with the hagedorn spectrum $\rho = \rho_{exp}$ as in Eq.(11). The dotted line correspond to taking discrete spectrum for hadron resonance gas. The right panel shows the same quantities but with the spectral function $\rho = \rho_{power}$ as given in Eq.(12). The data points are from the lattice simulation results taken from Ref. [39].

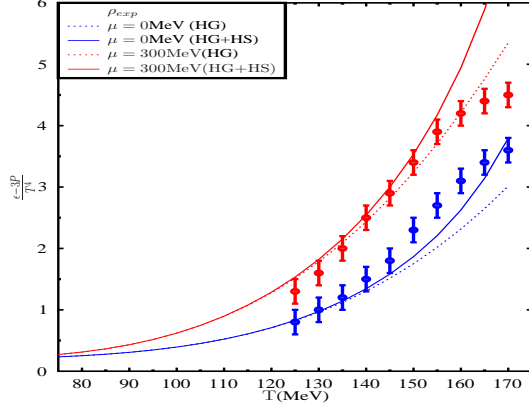
IV. RESULTS AND DISCUSSIONS

Let us first discuss the thermodynamics of hadron resonance gas specified by the spectral density as given by Eq.(10). To estimate different thermodynamic quantities, for the discrete part of spectrum, we have taken all the hadrons and their resonances with mass less than 2 GeV [44]. For the Hagedorn part, consider both the forms of spectral density given by Eq. (11) and Eq.(12).

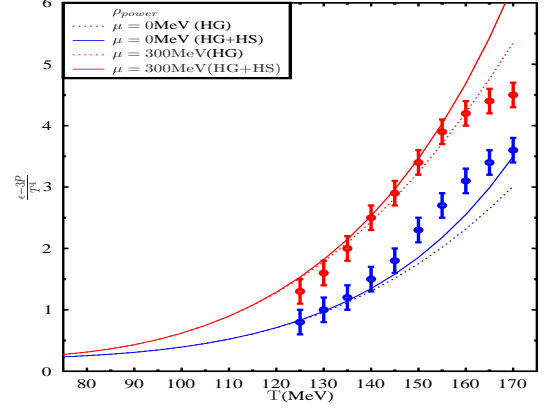
In Fig.1 we have plotted the pressure in units of T^4 for two different chemical potentials, $\mu = 0$ MeV and $\mu = 300$ MeV. The lattice points with the error bars have been taken from the table 4 of the Ref. [39] corresponding to the continuum extrapolation. The dotted lines in Fig.1 correspond to considering only the discrete part of the spectral density in Eq.(9). Left panel correspond to exponential form of spectral density for continuum part while right panel corresponds to power law form of spectral density in Eq.(10). As can be noted in this figure, the discrete spectrum coupled with continuum spectrum describe the lattice data quite well up to $T = 170$ MeV with the parametrization given in table 1 within the error bars of the lattice simulations.

In fig. 2 we have plotted the dimensionless scale anomaly $(\epsilon - 3p)/T^4$ as a function of temperature at two different chemical potentials. As can be noted from both the Figs (2a) and (2b), the discrete part of the spectral density does not give a good fit to the lattice data beyond 140 MeV, but when coupled with continuum part as in Eq.(10) gives good fit to lattice data up to 150 MeV even at $\mu = 300$ MeV reasonably well. We have taken a higher T_H value compared to [20] that was required to fit the lattice data [39]. This is because, in Ref.[20], the lattice data was taken for $N_t = 10$ lattice data of Ref.[6] while we have fitted with the continuum extrapolation of for $\mu = 0$ the lattice data in Ref.[39].

Fig 3 shows speed of sound squared (C_s^2) as a function of temperature at fixed values of chemical potential along with the lattice simulation results of Ref.[39]. As can be noted from the figure, keeping only the discrete part of the spectral density, does not fit the lattice results although the same could fit the lattice result for pressure and the scale anomaly results of Ref.[39]. On the other hand the power law parametrization for the continuum part of spectral density along with the discrete part leads to a reasonable fit to lattice data up to 150 MeV both at $\mu = 0$ and $\mu = 300$ MeV. The initial rise in sound velocity with temperature is reflection of the fact that the light degrees of freedom are excited easily at low temperature and contribute to pressure and energy. But at larger temperatures when baryons are excited, they contribute significantly to energy density but almost nothing to pressure. This leads to decrease of sound velocity with temperature seen at higher temperatures ($T > 80$ MeV). As chemical potential increases, heavier baryonic channels opens up at low temperature and contribute to energy density significantly but

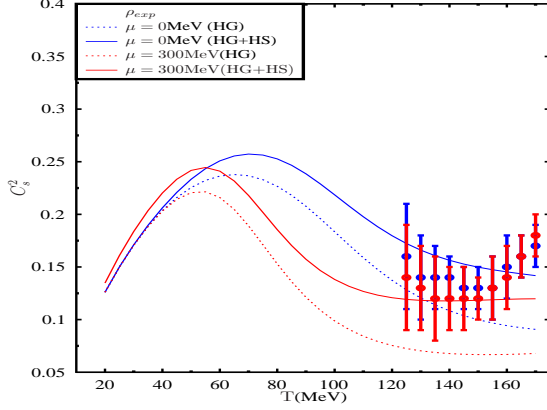


(2 a)

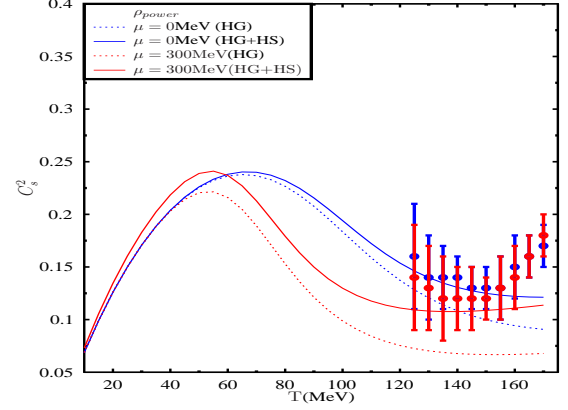


(2 b)

FIG. 2. Scale anomaly as a function of temperature for exponential spectral density (2 a) and power law spectral density function (2 b).



(3 a)



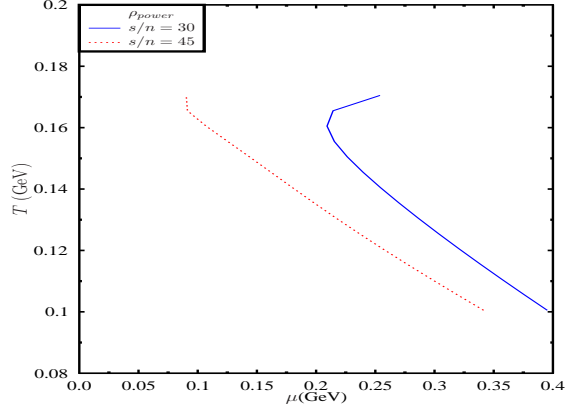
(3 b)

FIG. 3. Square of sound velocity as a function of temperature for exponential spectral density (3 a) and power law spectral density function (3 b).

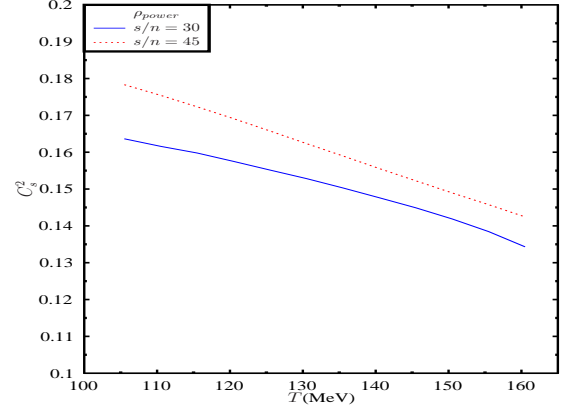
nothing to pressure. This leads to lower values of C_s^2 as the chemical potential is increased.

We have also plotted speed of sound for isentropic situation in figure 4. To get the chemical potential for a given temperature, we vary chemical potential so that ratio S/N is constant. Resulting isentropic trajectories in the $\mu - T$ phase space is shown in fig. (4a). $S/N = 30$ and $S/N = 45$ corresponds to AGS and SPS [45]. As expected from the results for constant chemical potential (Fig.3), sound velocity is lower for lower S/N .

We next use these thermodynamic results for the hadron resonance gas to Eq.(2) and Eq.(5) to estimate the bulk viscosity. We also include here the contributions from the quark condensates in the discrete part of the spectrum using Eq.(18). Contribution of these terms to ζ/s turns out to be only few percent of the the total contribution. The resulting behavior of ζ/s as a function of temperature is shown in Fig.5 for different values of the baryon chemical potential. In general, the ratio decrease with temperature at low temperature followed by a sharp increase and finally

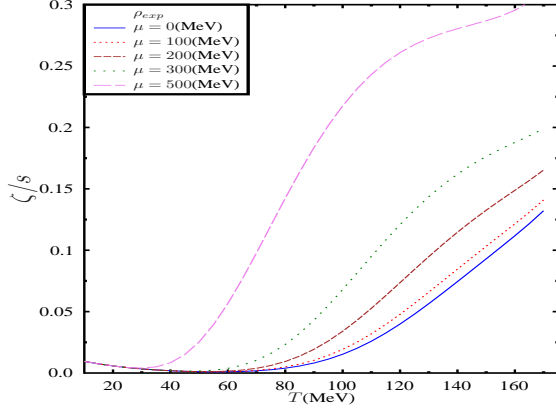


(4 a)

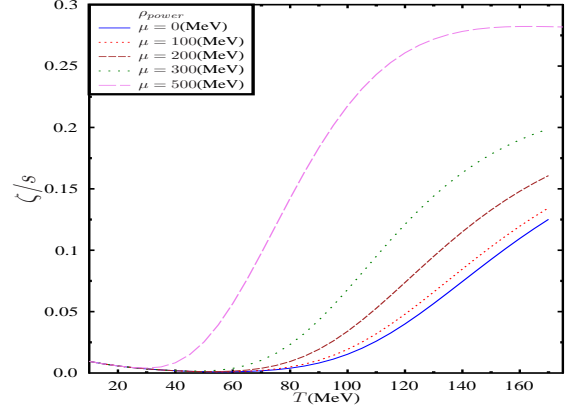


(4 b)

FIG. 4. Velocity of sound at constant entropy per baryon ratios. Left panel (4a) shows trajectories of constant entropy per baryon in the phase diagram. velocity of sound for constant entropy per baryon is plotted in Fig. 4b.



(5 a)

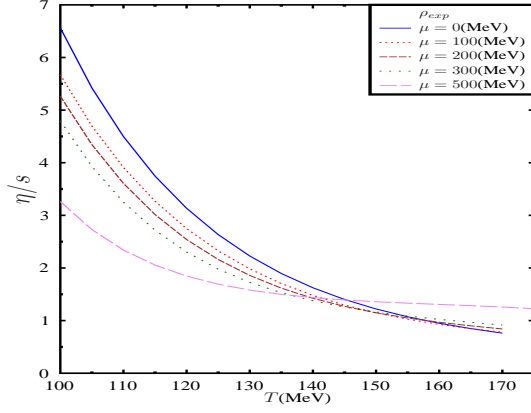


(5 b)

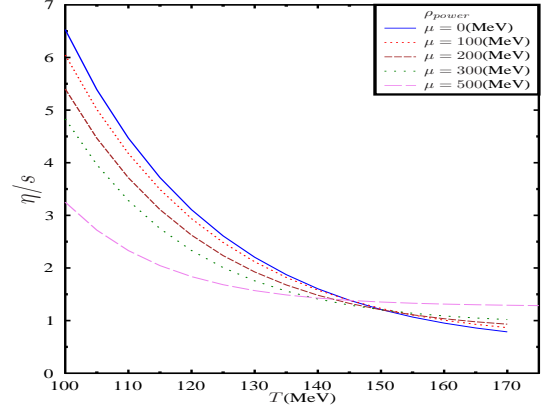
FIG. 5. Bulk viscosity to entropy ratio as a function of temperature for different chemical potentials. Left panel is with exponential hadron spectrum and the right panel is with power law hadron spectrum

flattens out at temperatures around 160 MeV. This behavior is connected with the behavior of velocity of sound with temperature through Eq.(5). The initial decrease of ζ/s with temperature is due to increase of sound velocity at low temperature due to excitation of light hadrons. At temperature $T > 60$ MeV, the sharp rise is related to the decrease of velocity of sound with excitations of heavier hadrons leading to decrease of sound velocity which finally flattens out at temperatures around 155 MeV as shown in Fig.3. The larger bulk viscosity to entropy ratio at higher chemical potential is again related to decrease of velocity of sound due to excitation of heavier baryons.

In Fig.6, we have plotted the shear viscosity to entropy ratio for different chemical potentials as a function of temperature. The finite volume effects arise here through the $\frac{1}{r^2}$ factors arising from the finite size of the hadrons as in Eq.(7). We also retain here the finite volume corrections to the entropy density s as in Ref.[46]. We might mention here that the thermodynamic quantities are not sensitive to the value of r for $r > 0.2$ fm as was demonstrated in

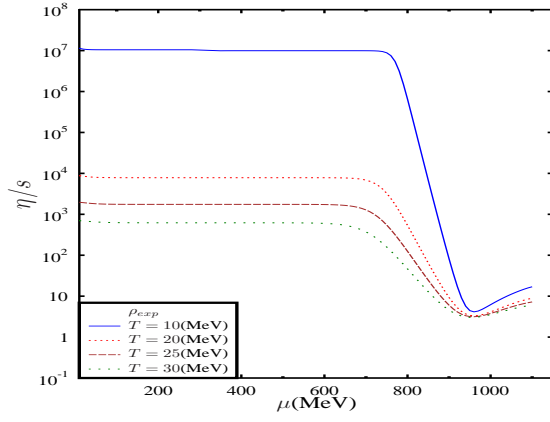


(6 a)

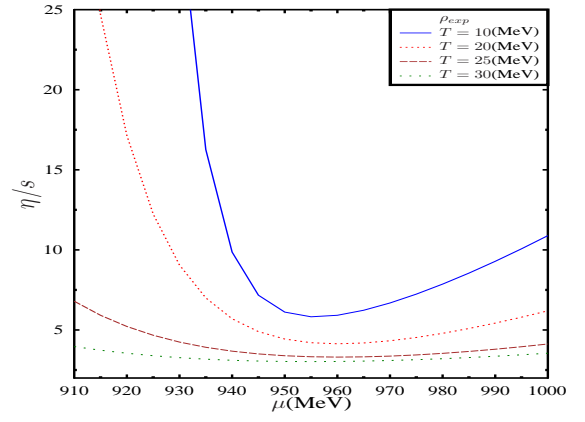


(6 b)

FIG. 6. Shear viscosity to entropy ratio in the hadronic phase. Left panel (6 a) shows $\frac{\eta}{s}$ as a function of temperature for different chemical potential with the exponential hagedorn spectrum. The right panel shows the same with the power law hagedorn spectrum.



(7 a)



(7 b)

FIG. 7. Shear viscosity to entropy ratio as a function of chemical potential.

Ref.[20]. We have taken here a uniform size of $r = 0.4 fm$ for all mesons and $r = 0.5 fm$ for all baryons [17, 47]. For $\mu = 0$ the minimum reaches about $\frac{\eta}{s} = 0.7$ which is an order of magnitude larger than viscosity bound of $\frac{\eta}{s} = \frac{1}{4\pi}$. As the chemical potential is increased, temperature dependence is similar to that at $\mu = 0$. On the other hand, the behavior of the ratio η/s is nontrivial. It decreases with chemical potential for temperature less than about 130 MeV, beyond which increases with μ . The initial decrease of η with respect to μ can be understood as enhancement of the hardcore cross section with nucleon number density. However, the entropy density σ starts decreasing with increase in chemical potential at higher temperature. This is due to the fact that the volume corrections proportional to the density of the particles enter in the denominator for the entropy density [46]. This in turn makes a larger value for the ratio $\frac{\eta}{s}$.

We also looked into the behavior of $\frac{\eta}{s}$ at low temperatures as a function of μ where it shows a valley structure

which is plotted in Fig.7. Such an observation was also made in Ref.s[24, 25, 49]. The existence of a minimum of η/s was interpreted in these references indicative of a liquid gas phase transition. This is due to the fact that a minimum in the ratio η/s as a function of the controlling parameter of thermodynamics like temperature or chemical potential could be indicative of a phase transition [24, 25, 48]. As the temperature increase, the valley structure become shallower as is clearly shown in Fig.7 b, possibly suggestive of the phase transition. However, on the other hand, the corresponding entropy does not show such a structure. Further, the corresponding nucleon number density here ($0.07/fm^3$), however, turns out to be about half the nuclear matter density.

V. SUMMARY

We have here tried to estimate the bulk and shear viscosity to entropy ratio in a hadronic medium modeling the same as a hadron resonance gas. Apart from including all the hadrons below a cutoff of 2 GeV, we have also included a continuum density of state beyond 2GeV. Such a description of hadronic model gives a good fit to the lattice data both at zero and finite chemical potential [39]. The thermodynamic quantities so obtained is used to estimate the bulk viscosity of hadron gas at finite chemical potential using the method as outlined in Ref.[11] for finite temperature and zero chemical potential. At finite chemical potential, the $\frac{\zeta}{s}$ become higher as compared to $\mu = 0$ and is related to the fact that the velocity of sound becomes smaller due to finite chemical potential with excitation of heavier baryons contributing more to the energy density as compared to the pressure.

This approach has already been used to estimate η/s for hadronic medium in Ref. [18] to obtain η/s reaching the viscosity bound of $\frac{1}{4\pi}$ at temperature of about $T = 190\text{MeV}$ using the lattice data available at that time. However, later lattice data pointed to a lower critical temperature giving rise to indicate that the hadron resonance gas can lead to η/s being about an order of magnitude higher than the viscosity bound. We observe that at finite chemical potential $\frac{\eta}{s}$ increases with temperature with its magnitude increasing with chemical potential. For low temperatures ($T < 30\text{MeV}$) and high baryon chemical potential, we observed a valley structure for this ratio which can have a connection with liquid gas phase transition in nuclear matter.

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